

ER622186095

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

APPLICATION FOR LETTERS PATENT

**SYSTEMS AND METHODS FOR ENCODING
RANDOMLY DISTRIBUTED FEATURES IN AN
OBJECT**

Inventor(s):
Darko Kirovski

ATTORNEY'S DOCKET NO. MS1-1934US

1

2 **TECHNICAL FIELD**

3

4 The systems and methods described herein generally relate to counterfeit-
5 resistant and/or tamper-resistant labels, and more particularly, to utilizing
6 randomly distributed features of an object (whether embedded or naturally
7 inherent) to limit unauthorized attempts in counterfeiting and/or tampering with
the label.

8

9 **BACKGROUND OF THE INVENTION**

10 Counterfeiting and tampering of labels cost product marketers and
11 manufacturers billions of dollars each year in lost income and lost customers.
12 With the proliferation of computer technology, generating labels that resemble the
13 genuine item has become easier. For example, a scanner may be utilized to scan a
14 high-resolution image of a genuine label which can then be reproduced repeatedly
15 at a minimum cost. Also, coupons may be scanned, modified (e.g., to have a
16 higher value), repeatedly printed, and redeemed.

17 Various technologies have been utilized to stop the flood of counterfeiting
18 and tampering in the recent years. One way labels have been secured is by
19 incorporation of bar codes. Bar codes are generally machine-readable code that is
20 printed on a label. Using a bar code scanner, the label with a bar code may be
21 quickly read and authenticated. One problem with current bar coded labels is that
22 an identical label may be used on various items.

23 Another current solution is to have the scanned bar code examined against
24 secure data stored in a database (e.g., a point of sale (POS) system). This solution,
25 however, requires incorporation of up-to-date data from a marketer or

1 manufacturer. Such a solution requires timely and close cooperation of multiple
2 entities. Also, such a solution limits its implementation flexibility and may not
3 always be feasible.

4 These technologies, however, share a common disadvantage; namely, the
5 labels scanned are physically identical for a given product. Accordingly, even
6 though the manufacturing process for creating the legitimate labels may be highly
7 sophisticated, it generally does not take a counterfeiter much time to determine a
8 way to create fake pass-offs. And, once a label is successfully copied a single
9 time, it may be repeatedly reproduced (e.g., by building a master copy that is
10 replicated at low cost). Even if a label is black-listed in a database after a given
11 number of uses, there is no guarantee that the labels that are scanned first are
12 actually the genuine labels.

13 Accordingly, the current solutions fail to provide labels that are relatively
14 hard to copy and inexpensive to produce.

15
16 **SUMMARY OF THE INVENTION**

17 The systems and methods described herein are directed at encoding
18 randomly distributed features in an object. In one aspect, randomly distributed
19 features in an authentication object are determined. Data representing the
20 randomly distributed features is compressed and encoded with a signature. A
21 label is created and includes the authentication object and the encoded data.

22 In another aspect, the data is compressed by determining a probability
23 density function associated with the authentication object. Vectors associated with
24 the randomly distributed attributes are determined based, at least in part, on the
25

1 probability density function. The vectors are encoded using an arithmetic coding
2 algorithm.

3 **BRIEF DESCRIPTION OF THE DRAWINGS**

4
5 Fig. 1 shows an example authentication object for use as part of a label,
6 such as a certificate of authenticity.

7 Fig. 2 is a schematic diagram illustrating an example certificate of
8 authenticity system and example procedures employed by the system for issuing
9 and verifying a certificate of authenticity.

10 Fig. 3A is a schematic diagram of an example scanning system for
11 capturing randomly distributed features of an authentication object associated with
12 a certificate of authenticity.

13 Fig. 3B is a top view of the authentication object shown in Fig. 3A.

14 Fig. 4 is a flow diagram of an example process that may be used to create a
15 certificate of authenticity.

16 Fig. 5 is a flow diagram of an example process that may be used to
17 compress data that represents the randomly distributed attributes of an
18 authentication object.

19 Figure 6 is a graphical representation of areas that correspond to four
20 different regions in an example authentication object.

21 Figure 7 is a graphical representation of the nineteen different regions on an
22 example authentication object.

23 Fig. 8 is a graph of an example of the probability density function for a
24 square authentication object.

25 Figure 9 is a graphical representation of areas in an authentication object.

1 Fig. 10 is a graphical representation of an example of how an arithmetic
2 coder encodes the string "aba".

3 Figure 11 is an example of an instance of an authentication object shown
4 with nodes.

5 Figure 12 is a graphical representation of a certificate of authenticity
6 designed for optimizing cost effectiveness.

7 Fig. 13 illustrates an example computing device which the described
8 systems and methods can be either fully or partially implemented.

9

DETAILED DESCRIPTION

10

I. Introduction

11 The systems and methods described herein are directed at encoding
12 information about the randomly distributed features of an object used in a label.
13 Labels may include any type of identification means that are attached to or
14 incorporated within an item. A label that is configured to be authenticated is
15 referred herein as a certificate of authenticity. An object with randomly
16 distributed features used in a certificate of authenticity is referred to herein as an
17 authentication object. To enable self-authentication, a certificate of authenticity
18 may include both the authentication object and the information about the randomly
19 distributed features. A compression method may be used to increase the amount
20 of information about the randomly distributed features that can be encoded and
21 included in the certificate of authenticity. According to one example calculation,
22 the cost of forging a certificate of authenticity is exponentially increased
23 proportional to the improvement in compressing the information. This substantial
24

1 increase in forging cost results in a reliable certificate of authenticity that is
2 relative cheap to manufacture but is difficult to falsify.

3 Fig. 1 shows an example authentication object 100 for use as part of a label,
4 such as a certificate of authenticity. To be effectively used in a certificate of
5 authenticity, authentication object 100 typically contains randomly distributed
6 features that are unique and are hard to replicate. The example authentication
7 object 100 shown in Fig. 1 is part of a fiber-based certificate of authenticity and
8 contains fibers 110 that are embedded in the object in a random manner. Fibers
9 110 serve as the randomly distributed features of authentication object 100. Fibers
10 110 may be incorporated in authentication object 100 by any means. For example,
11 fibers 100 may be sprayed onto authentication object 100. Fibers 100 may also be
12 embedded into authentication object 100 during the manufacturing process. In one
13 embodiment, fibers 110 are optical fibers capable of transmitting light between
14 their endpoints. Thus, by shedding light on a certain region 120 of authentication
15 object 100, endpoints of fibers 131-133 that have at least one end-point within the
16 lit up region are illuminated.

17 In Fig. 1, authentication object 100 includes κ randomly distributed fibers.
18 Authentication object 100 may be scanned at a resolution of $L \times L$ pixels. Each
19 fiber has a fixed length of R . Although the example authentication object 100 in
20 Fig. 1 contains fibers, it is to be understood that authentication objects with other
21 randomly distributed features may also be used in a certificate of authenticity in a
22 similar manner.

23 The randomly distributed features of authentication object 100 may be
24 used in a certificate of authenticity to protect the proof of authenticity of an
25 arbitrary object, such as a product. For example, certain hard-to-replicate data

1 about the randomly distributed features of the certificate of authenticity may be
2 digitized, signed with the private key of the issuer, and the signature may be
3 imprinted on the certificate of authenticity in a machine-readable form to validate
4 that the produced instance is authentic. Each instance of the certificate of
5 authenticity is associated with an object whose authenticity the issuer wants to
6 vouch. In one embodiment, verification of authenticity is done by extracting the
7 signed data (data about the randomly distributed features) using the public key of
8 the issuer and verifying that the extracted data matches the data of the associated
9 instance of the certificate of authenticity. In order to counterfeit protected objects,
10 the adversary needs to either: (i) figure out the private key of the issuer, (ii) devise
11 a manufacturing process that can exactly replicate an already signed instance of
12 the certificate of authenticity, or (iii) misappropriate signed instances of the
13 certificate of authenticity. From that perspective, the certificate of authenticity can
14 be used to protect products whose value roughly does not exceed the cost of
15 forging a single certificate of authenticity instance, including the accumulated
16 development of a successful adversarial manufacturing process.

17 A goal of a certificate of authenticity system is to ensure the authenticity of
18 products or certain information associated with a product. The set of applications
19 is numerous and broad, ranging from software and media (e.g., DVD, CD) anti-
20 piracy to unforgeable coupons and design of tamper-proof hardware. For
21 example, creating a tamper-resistant chip would require coating its package with a
22 certificate of authenticity. Before each usage, the integrity of the certificate of
23 authenticity should be verified in order to verify authenticity of the protected
24 silicon.

1 Below, example hardware platforms for inexpensive but efficient read-out
2 of the randomly distributed features of a fiber-based certificate of authenticity will
3 be discussed. The hardware platforms may include a barcode. Since the capacity
4 of a barcode for low-cost readers is limited to about 3K bits, the message signed
5 by the private key is limited to the same length. Also, since one of the goals of a
6 certificate of authenticity system is to maximize the effort of the adversary who
7 aims at forging a specific instance of the certificate of authenticity, the problem
8 associated with storing in the fixed-length signed message as much as possible
9 information about the unique and randomly distributed features of a fiber-based
10 certificate of authenticity will be discussed. An example analytical model for a
11 fiber-based certificate of authenticity will be provided. Then, the discussion
12 below will also formalize the problem of compression of a point set, and show that
13 optimal compression of fibers' positions in an instance of a certificate of
14 authenticity is an NP-complete problem. In order to heuristically address this
15 problem, an algorithm which significantly improves upon compression ratios of
16 conventional compression methodologies will be provided.

17

18 II. Issuing and Verifying Certificate of Authenticity

19 Fig. 2 is a schematic diagram illustrating an example certificate of
20 authenticity system 200 and example procedures employed by the system for
21 issuing and verifying a certificate of authenticity. Certificate of authenticity
22 system 200 includes certificate of authenticity 210, an issuer 230, and a verifier
23 250. As shown in Fig. 2, certificate of authenticity 210 may include the
24 authentication object 100 in Fig. 1, a barcode 213, and text 215.

1 The information that needs to be protected on a certificate of authenticity
2 includes: (a) the representation of the hard-to-replicate randomly distributed
3 features of authentication object 100 and (b) an arbitrary associated textual data.
4 Initially, the randomly distributed features of authentication object 100, such as
5 locations of fibers, are scanned using a hardware device. Details on how this
6 information is collected and represented will be discussed below in conjunction
7 with Fig. 3.

8 For the purpose of discussion, assume that the resulting information f is a
9 random string of n_F bits. Parameter n_F is fixed and equals $n_F = k * n_{RSA}$, $k \in N$,
10 where n_{RSA} is the length of an RSA public-key (for example, $n_{RSA} = 1024$) and k is
11 commonly set to $k \in [1, 3]$. Given a fixed n_F , the digest f of data 231 representing
12 the randomly distributed features of authentication object 100 may statistically
13 maximize the distance between any two distinct certificate of authenticity
14 instances. This goal translates directly to minimized likelihood of a false negative
15 and false positive during the verification step.

16 The textual data t is an arbitrary string of characters which depends on the
17 application (e.g., expiration date, manufacturer's warranty). The textual data is
18 derived from text 215, which is printed on certificate of authenticity 210 as shown
19 in Fig. 2.

20 The textual data may be hashed using a cryptographically secure hash
21 algorithm 237, such as SHA1. The output of the hash function is denoted as a
22 message t with n_T bits. Issuer 230 creates the message m that may be signed by
23 RSA. For example, messages f and t are merged into a message m of length
24 $n_M = n_F$ using a reversible operator \otimes that ensures that each bit of m is dependent
25 upon all bits from both f and t . This step may maximize the number of bits that

1 need to be manipulated in data 231 as well as text 215 to create a certain message
2 m . An example of such an operator is symmetric encryption $m = t \otimes f \equiv E_t(f)$ of
3 f using t or certain subset of bits from t as a key. Message m is signed with an
4 RSA signature 235 using the private-key 233 of the issuer 230. Each n_{RSA} bits of
5 m are signed separately. The resulting signature s has $n_s = n_M = n_F$ bits. This
6 message is encoded and printed as barcode 213 (such as barcodes that obey the
7 PDF417 standard) onto certificate of authenticity 210.

8 The verification of certificate of authenticity 210 involves several steps.
9 Verifier 250 initially scans the printed components: text 215 and barcode 213.
10 Barcode 213 is decoded into the originally printed signature s . Text 215 is
11 scanned and is hashed in order to create the message t . Note that generic optical
12 character recognition (OCR) is not required for this task because the font used to
13 print the text is known to the verifier 250 and optimized for improved OCR. For
14 successful certificate of authenticity verification, text 215 and barcode 213 need to
15 be read without errors; a task which is readily achievable with modern scanning
16 technologies.

17 Verifier 250 performs the RSA signature verification 255 on s using
18 issuer's public-key 253 and obtains the signed message m . Verifier 250 can then
19 compute $f = m(\otimes)^{-1}t$. In the example of using encryption as \otimes , this is achieved
20 via decryption $f = E_t^{-1}(m)$. Next, verifier 250 scans data 251 of representing the
21 randomly distributed features in authentication object 251 and creates their
22 presentation f' . Verifier 250 compares f' to the extracted f . Verifier 250 needs
23 to quantify the correlation between the two sets of data: the one attached to the
24 certificate and the one used to create the signature on the certificate of
25 authenticity. At decision block 259, if the level of similarity of the two sets of

1 data surpasses a certain threshold, verifier 250 announces that the certificate of
2 authenticity 210 is authentic and vice versa.

3 Fig. 3A is a schematic diagram of an example scanning system 300 for
4 capturing randomly distributed features of authentication object 310 associated
5 with a certificate of authenticity. Scanning system 300 includes optical sensor
6 322 and light source 324. Optical sensor 322 is configured to scan authentication
7 object 310 and may include a charged coupled device (CCD) matrix of a particular
8 resolution. In one embodiment, optical sensor 322 has a resolution of 128 x 128
9 pixels. Light source 324 is configured to provide light of a particular wavelength
10 to illuminate a region of authentication object 310. Light source 324 may include,
11 for example, a light emitting diode (LED). As shown in Fig. 3A, one end of fiber
12 326 in authentication object 310 is illuminated by light source 324. The light is
13 transmitted to the other end of fiber 326 and is sensed by optical sensor 322.

14 Fig. 3B is a top view of the authentication object 310 in Fig. 3A. In
15 operation, the scanning system 300 divides authentication object 310 into regions,
16 such as regions 311-314. As shown in Fig. 3B, light source 324 of scanning
17 system 300 sheds light onto region 314 while regions 311-313 are isolated from
18 light source 324. By illuminating region 314, the location of the endpoints in
19 regions 311-313 of authentication object 310 can be determined by optical sensor
20 322. Thus, the read-out of the randomly distributed features in authentication
21 object 310 includes four digital images that contain four different point-sets. Each
22 point-set is associated with a particular region and is determined by illuminating
23 that region.

24 It is conceivable that advancement in technology, such as nanotechnology,
25 may enable an electronic device to decode the randomly distributed features from

1 a certificate of authenticity and create a light pattern that corresponds to these
2 features. Such a device may be able to forge the certificate of authenticity. In one
3 embodiment, scanning system 300 may be configured to prevent this method of
4 forging by changing the wavelength (e.g. color) of the light used by light source
5 324. For example, the wavelength of the light may be randomly selected each
6 time an authentication object is scanned by scanning system 300. Optical sensor
7 322 may be configured to detect the wavelength of the light emitted by the fibers
8 in the authentication object and to determine whether that wavelength corresponds
9 to the wavelength of the light emitted by light source 324. If the wavelengths of
10 the emitted and detected light do not match, the certificate of authenticity is likely
11 a forgery.

12 Fig. 4 is a flow diagram of an example process 400 that may be used to
13 create a certificate of authenticity. At block 405, the authentication object in a
14 certificate of authenticity is scanned. The authentication object may be scanned
15 using scanning system 300 in Fig. 3A.

16 At block 410, data representing the randomly distributed attributes of the
17 authentication object is determined. In a fiber-based authentication object, the
18 data may include the positions of the endpoints of fibers that are illuminated, such
19 as the endpoints shown in Fig. 3B.

20 At block 415, the data is compressing to enhance the security level of the
21 certificate of authenticity. Data compression will be discussed in detail in
22 conjunction with Fig. 5. Briefly stated, a path may be determined for compressing
23 a portion of the data representing randomly distributed attributes in the
24 authentication object.

1 At block 420, the compressed data is encoded. For example, the
2 compressed data may be signed using private-key 233 in Fig. 2. At block 425, the
3 encoded data is incorporated in the certificate of authenticity. For example, the
4 encoded data may be printed onto the certificate of authenticity as a barcode, such
5 as barcode 213 in Fig. 2.

6 Fig. 5 is a flow diagram of an example process 500 that may be used to
7 compress data that represents the randomly distributed attributes of an
8 authentication object. For the purpose of discussion, process 500 will be described
9 in the context of a fiber-based certificate of authenticity. However, process 500
10 may be applied to any type of certificate of authenticity.

11 At block 505, a probability density function associated with the
12 authentication object is determined. Probability density function will be discussed
13 in Section III-A. An example probability density function is shown in Equation
14 11. A graphical presentation of the example probability density function is
15 illustrated in Fig. 8. Briefly stated, the probability density function represents the
16 likelihood that a unit of the randomly distributed attributes is found in a certain
17 location of the authentication object. In the context of a fiber-based certificate of
18 authenticity, the probability density function may represent the probability that a
19 particular point in a region of the authentication object is illuminated. The
20 probability density function may also be used to compute how many of the total
21 fibers will be illuminated in a particular region.

22 At block 510, vectors associated with the randomly distributed attributes
23 are determined. In the context of a fiber-based certificate of authenticity, point-to-
24 point vectors are used and will be discussed in Section IV-A. In particular,
25

1 Equation 16 may be used to compute point-to-point vectors to represent the
2 randomly distributed attributes in a fiber-based certificate of authenticity.

3 At block 515, the vectors are encoded using an arithmetic coding algorithm.
4 Arithmetic coding algorithm will be discussed in Section IV-A. An example
5 algorithm is shown in Table 2.

6 At block 520, a path for compressing a portion of the vectors within a fixed
7 amount of data is determined. The method for computing the path is discussed in
8 Section IV-B. The example path may be computed using Equation 20. At block
9 525, the path of the compressed data representing a portion of the randomly
10 distributed attributes is returned.

11 12 **III. Certificate of Authenticity Model**

13 In this section, an analytical model of a fiber-based certificate of
14 authenticity is discussed. Two features of a certificate of authenticity S are
15 modeled. Given that a particular region S_i of the certificate of authenticity is
16 illuminated, the probability density function that a particular point in $S - S_i$ is
17 illuminated is computed. Also, given that K fibers are in S , the expected number
18 of fibers that are illuminated in $S - S_i$ is also computed.

19 20 **A. Distribution of Illuminated Fiber End-Points**

21 An authentication object (L,R,K) is modeled as a square with an edge of L
22 units and K fibers of fixed length $R \leq L/2$ randomly thrown over the object. Other
23 model variants, such as variable fiber length or arbitrary shape authentication
24 object, can be derived from this model. The authentication object is positioned in
25 the positive quadrant of a 2D Cartesian coordinate system as illustrated in Fig. 1.

1 In addition, the authentication object is divided into four equal squares
 2 $S = \{S_1, S_2, S_3, S_4\}$. Each of them is used to record the 3D fiber structure as
 3 described above in conjunction with Fig. 3A and 3B. Next, a fiber is denoted as a
 4 tuple $f = \{A, B\}$ of points $A, B \subset S$ such that the distance between them is
 5 $\|A - B\| = R$.

6
 7 **Definition 1. Distribution of Illuminated Fiber End-Points.** Given that
 8 one of the squares S_i is illuminated, the probability density function (pdf)
 9 $\varphi(i, Q(x, y))$ is defined for any point $Q(x, y) \subset S - S_i$ via the probability $\xi(i, P)$ that
 10 any area $P \subset S - S_i$ contains an illuminated end-point A of a fiber $f = \{A, B\}$,
 11 conditioned on the fact that the other end-point B is located in the illuminated
 12 region S_i . More formally, for any $P \subset S - S_i$:

$$13 \quad \xi(i, P) = \Pr[A \subset P \mid f = \{A, B\} \subset S, B \subset S_i] \quad (6)$$

$$14 \quad = \iint_{Q(x, y) \subset P} \varphi(i, Q(x, y)) dx dy.$$

17
 18 Assume that throwing a fiber $f = \{A, B\}$ into an authentication object
 19 consists of two dependent events: (i) first end-point A lands on the authentication
 20 object and (ii) second end-point B hits the authentication object. While A can
 21 land anywhere on the COA, the position of B is dependent upon the location of
 22 A . Endpoint B must land on part of the perimeter of the circle centered around A ,
 23 with a radius R , and contained within the authentication object. In the remainder
 24 of this subsection, the function $\varphi(i, Q(x, y))$ is analytically computed based on the
 25

1 analysis of the events (i-ii). For brevity, only $\varphi(1, Q(x, y))$ is computed for the case
2 when region S_1 is lit up. $\varphi(1, Q(x, y))$ are computed in two steps.

3

4 **Definition 2. Perimeter Containment.** First, for a given point $A \subset S$, the
5 perimeter containment function $\varrho(A)$ is defined, which measures the length of the
6 part of the perimeter (arc) of the circle centered at A with radius R that is
7 encompassed by the entire authentication object S . There are four different
8 regions in the authentication object (marked P1 through P4 in Fig. 6) where $\varrho(A)$
9 is uniformly computed.

10 Fig. 6 is a graphical representation of areas P1-P4 that correspond to the
11 four different regions in an example authentication object 600. For each point in a
12 certain area PX , the perimeter containment function is computed using a closed
13 analytical form distinct for that area using Equations 7-10 as discussed below.

14 AREA P1. This is the central area of the authentication object, where for any
15 point $Q \subset P1$, the circle with radius R centered at Q does not intersect with any of
16 the edges of the authentication object. The area is bounded by: $R \leq x \leq L - R$,
17 $R \leq y \leq L - R$.

18

$$\varrho(Q(x, y)) = 2R\pi. \quad (7)$$

19

20 AREA P2. There are four different P2 regions, where a circle with radius R
21 centered at any point $Q \subset P2$ intersects twice with exactly one edge of the
22 authentication object. For brevity, consideration is give only for the following one:
23 $R \leq x \leq L - R$, $0 \leq y < R$. Equations for other three regions can be symmetrically
24 computed.

25

$$\varrho(Q(x, y)) = R \left[\pi + 2 \arcsin \left(\frac{y}{R} \right) \right]. \quad (8)$$

AREA P3. There are four different P3 regions, where a circle with radius R centered at any point $Q \subset P3$ intersects twice with two different edges of the authentication object. Consideration is give only for the following one: $0 \leq x < R$, $0 \leq y < R$, $x^2 + y^2 \geq R^2$.

$$\varrho(Q(x, y)) = 2R \left[\pi - \arccos \left(\frac{x}{R} \right) - \arccos \left(\frac{y}{R} \right) \right]. \quad (9)$$

AREA P4. There are four different P4 regions, where a circle with radius R centered at any point $Q \subset P4$ intersects once with two edges of the COA. Consideration is give only for the following one: $x^2 + y^2 < R^2$.

$$\varrho(Q(x, y)) = R \left[\frac{\pi}{2} + \arcsin \left(\frac{x}{R} \right) + \arcsin \left(\frac{y}{R} \right) \right]. \quad (10)$$

In all Equations 8-10, only the return values of functions $\arcsin(\cdot)$ and $\arccos(\cdot)$ that are within $\{0, \pi/2\}$ are considered.

In the second step, the actual $\varphi(l, Q(x, y))$ is computed based on the fact that an illuminated endpoint A of a fiber $f = \{A, B\}$ is at position $A = Q(x, y)$ only if B is located on the part(s) of the circle $C(Q, R)$ centered at $Q(x, y)$ with a diameter R and contained by S_1 .

1 **Lemma 3. Dependence of $\varphi(i, Q(x, y))$ from $\varrho(Q(x, y))$.** Using function
 2 $\varrho(Q(x, y))$, pdf $\varphi(i, Q(x, y))$ is computed using the following integral:

$$3 \quad 4 \quad \varphi(i, Q(x, y)) = \int_{C(Q, R) \subset S_i} \frac{\alpha R d\vartheta}{\varrho(Q(x + R \cos \vartheta, y + R \sin \vartheta))}. \quad (11)$$

6 where ϑ browses the perimeter of $C(Q, R) \subset S_i$ and α is a constant such
 7 that:

$$9 \quad 10 \quad \iint_{Q(x, y) \subset S - S_i} \varphi(i, Q(x, y)) dx dy = 1. \quad (12)$$

11 A point $Q \subset S - S_i$ can be illuminated only due to a fiber $f = \{Q, B\}$, such
 12 that $B \subset S_i$. This implicates that B is located somewhere on the perimeter of the
 13 circle $C(Q, R)$ contained by S_i . For a given fiber $f = \{A, B\}$, the probability that A
 14 lands on a specific infinitesimally small arc of length $dl \subset S$, is equal to $dl/\varrho(B)$.

15 Hence:

$$17 \quad 18 \quad \varphi(i, Q) = \text{area}(S - S_i)^{-1} \int_{C(Q, R) \subset S_i} \frac{4 R d\vartheta}{\varrho(B(Q, R, \vartheta) \subset C) dl}, \quad (13)$$

20 where function $\text{area}(S - S_i)$ computes the area under $S - S_i$. Thus, the pdf
 21 $\varphi(i, Q(x, y))$ at a point $Q \subset S - S_i$ is proportional to the integral of the inverse of the
 22 value of $\varrho(\cdot)$ over $C(Q, R) \subset S_i$.

23 Figure 7 is a graphical representation of the nineteen different regions on an
 24 example authentication object 700 that have distinct analytical formulae as a
 25 solution to the integral quantified in Equation 11. For brevity, $\varphi(i, Q(x, y))$ is

1 approximately solved using a simple numerical computation. The results is
 2 illustrated in Fig. 8

3 Fig. 8 is a graph of an example probability density function for a square
 4 authentication object with parameters $L = 64$ and $R = 28$ sampled at unit points.
 5 Fig. 8 shows that the likelihood that an endpoint of a fiber lands on a certain small
 6 area $P \subset S - S_1$ varies significantly depending on the particular position of P
 7 within $S - S_1$. By using the information about the variance of $\varphi(i, Q(x, y))$
 8 throughout $S - S_1$, the point-subset compression algorithms can be significantly
 9 improved, as presented in Section IV. Manufacturing authentication object such
 10 that $\varphi(i, Q(x, y)) = \text{const.}$ over the entire area $S - S_1$, is a non-trivial task, probably
 11 as difficult as forging an original authentication object.

Area	Bounds	$\psi(1, Q(x, y))$
T0	$0 \leq x \leq L/2 - R, 0 \leq y \leq L/2 - R$	0
T1	$x^2 + (y - L/2)^2 < R^2, 0 \leq x \leq L/2 - R,$ $L/2 - R < y \leq L/2$	$R \left[\arcsin\left(\frac{x}{R}\right) + \arccos\left(\frac{L/2-y}{R}\right) \right]$
T2	$x^2 + (y - L/2)^2 \geq R^2, 0 \leq x \leq L/2 - R,$ $L/2 - R < y \leq L/2$	$2R \arccos\left(\frac{L/2-y}{R}\right)$
T3	$x^2 + (y - L/2)^2 \geq R^2,$ $(x - L/2)^2 + y^2 \geq R^2,$ $(x - L/2)^2 + (y - L/2)^2 \geq R^2$	$2R \left[\arccos\left(\frac{L/2-y}{R}\right) + \arccos\left(\frac{L/2-x}{R}\right) \right]$
T4	$x^2 + (y - L/2)^2 < R^2,$ $(x - L/2)^2 + y^2 < R^2,$ $(x - L/2)^2 + (y - L/2)^2 \geq R^2$	$R \left[\arcsin\left(\frac{x}{R}\right) + \arcsin\left(\frac{y}{R}\right) \right]$ $R \left[\arccos\left(\frac{L/2-y}{R}\right) + \arccos\left(\frac{L/2-x}{R}\right) \right] +$
T5	$x^2 + (y - L/2)^2 < R^2,$ $(x - L/2)^2 + y^2 < R^2,$ $(x - L/2)^2 + (y - L/2)^2 < R^2$	$R \left[\frac{\pi}{2} + \arcsin\left(\frac{x}{R}\right) + \arcsin\left(\frac{y}{R}\right) \right]$
T6	$x^2 + (y - L/2)^2 < R^2,$ $(x - L/2)^2 + y^2 \geq R^2,$	$R \left[\arcsin\left(\frac{x}{R}\right) + \arccos\left(\frac{L/2-y}{R}\right) \right] +$ $2R \arccos\left(\frac{L/2-x}{R}\right)$

	$(x - L/2)^2 + (y - L/2)^2 \geq R^2,$ $L/2 - R < x \leq L/2$	
T7	$x^2 + (y - L/2)^2 < R^2,$ $(x - L/2)^2 + y^2 \geq R^2,$ $(x - L/2)^2 + (y - L/2)^2 < R^2$	$R \left[\frac{\pi}{2} + \arcsin\left(\frac{x}{R}\right) + \arccos\left(\frac{L/2-x}{R}\right) \right]$
T8	$x^2 + (y - L/2)^2 \geq R^2,$ $(x - L/2)^2 + y^2 \geq R^2,$ $(x - L/2)^2 + (y - L/2)^2 < R^2$	$R \left[\frac{\pi}{2} + \arccos\left(\frac{L/2-y}{R}\right) + \arccos\left(\frac{L/2-x}{R}\right) \right]$

Table 1.

B. Illumination Ratio of Fiber End-Points

Definition 3. Illumination Ratio of Fiber End-Points. For an authentication object (L, R, K) and its illuminated region S_i , the illumination ratio λ is defined as a probability that a fiber $f = \{A, B\}$ has landed such that one of its end-points is in $B \subset S - S_i$ conditioned on the fact that the other end-point is in $A \subset S_i$:

$$\lambda = \Pr[B \subset S - S_i \mid f = \{A, B\}, A \subset S_i]. \quad (14)$$

Definition 4. Possibly Illuminated Arc. For any point $A \subset S_i$, a function $\psi(i, A(x, y))$ is defined that measures the length of the part of the perimeter of $C(A, R)$ contained by $S - S_i$.

Figure 9 is a graphical representation of the areas T0-T8, where $\psi(i, Q(x, y))$ is computed using distinct closed analytical forms. $\psi(i, Q(x, y))$ is analytically computed based on the analysis of the events (i-ii) from Section III-A. Similarly to Section III-A, only in the case when region S_i is lit up is computed. There are nine different regions in the COA (marked T0 through T8 in Fig. 9)

1 where $\psi(1, Q)$ is computed uniformly. The analytical closed forms for $\psi(1, Q)$
2 depending on the location of Q within S_i are given in Table 1.

3
4 **Lemma 4. Dependence of $\psi(1, Q(x, y))$, $\varrho(Q(x, y))$, and λ .** The
5 illumination ratio defined as in Def.3, can be computed as follows:

6
7
$$\lambda = \int_{Q(x,y) \subset S_i} \frac{\psi(i, Q(x, y))}{\varrho(Q(x, y))} dx dy. \quad (15)$$

8

9 A circle centered at a point $A \subset S$ with radius R is denoted as $C(A, R)$. For
10 each point $Q \subset S_i$, the likelihood that the other end-point B of a fiber $f = \{Q, B\}$
11 lands within $S - S_i$, equals the ratio of lengths of parts of the perimeter of $C(Q, R)$
12 contained by $S - S_i$ and S respectively. By integrating this ratio over all points
13 within S_i , Equation 15 is obtained.

14 Given an authentication object (L,R,K), using λ , computed by numerically
15 approximating Equation 15 and the closed forms for $\psi(1, Q)$ from Table 1, one can
16 compute the expected number of illuminated points in $S - S_i$ when S_i is
17 illuminated as $\lambda K/2$. For example, for an authentication object (64,28,100) the
18 resulting $\lambda \approx 0.74$, which means that on the average, the number of illuminated
19 endpoints in case S_i is illuminated, is about $0.74 \cdot 50 = 37$.

20 **IV. Compression of a Point-Subset in a COA**

21 The goal of the certificate of authenticity system is to ensure that the task of
22 manufacturing (i.e. forging) a specific authentication object instance as difficult as
23 possible. This goal is quantified as a demand for recording the positions of as
24 many as possible fibers of the authentication object. In the example compression
25

1 algorithm, the number of regions of authentication object equals four; hence, for
2 each region S_i , a quarter $n_M/4$ of bits in the signed message m is dedicated to
3 storing as many as possible fiber end-points illuminated in $S - S_i$ once light is shed
4 on S_i . Note that in general, not all illuminated points need to be stored; only the
5 largest subset of these points that can be encoded using $n_M/4$ bits.

6 In this section, a mechanism is described, which is configured to encode the
7 distance between two illuminated points in an authentication object. The
8 mechanism is based on arithmetic coding. Next, the problem of compressing as
9 many as possible fiber endpoints using a constant number of bits is formalized.
10 Finally, the discussion will show that this problem is NP-complete and a
11 constructive heuristic as a sub-optimal solution is presented.

12 13 A. Encoding Point-to-Point Vectors

14 In this subsection, how a vector defined by its starting and ending point is
15 encoded using a near-minimal number of bits is described. An additional
16 constraint is that the points in the considered area occur according to a given pdf.

17 18 1) *Arithmetic coding:*

19 An arithmetic coder (AC) converts an input stream of arbitrary length into a
20 single rational number within $[0,1]$. The principal strength of AC is that it can
21 compress arbitrarily close to the entropy. The discussion below shows how a word
22 "aba" is encoded given an alphabet with an unknown pdf of symbol occurrence.

23 Fig. 10 is a graphical representation of an example of how an arithmetic
24 coder encodes the string "aba" is encoded given an alphabet $L = \{a, b\}$ with an
25 unknown pdf of symbol occurrence. The example is illustrated in Fig. 10.

Initially, the range of the AC is reset to $[0,1]$ and each symbol in L is given an equal likelihood of occurrence $\Pr[a] = \Pr[b] = 1/2$. Thus, the AC divides its range into two subranges $[0,0.5]$ and $[0.5,1]$, each representing "b" and "a" respectively. Symbol a is encoded by constraining the range of the AC to the range that corresponds to this symbol, i.e., $[0.5,1]$. In addition, the AC updates the counter for the occurrence of symbol "a" and recomputes $\Pr[a] = 2/3$ and $\Pr[b] = 1/3$. In the next iteration, according to the updated $\Pr[a], \Pr[b]$, the AC divides its range into $[0.5,0.6667]$ and $[0.6667,1]$, each representing "b" and "a" respectively. When "b" arrives next, the AC reduces its range to the corresponding $[0.5,0.6667]$, updates $\Pr[a] = \Pr[b] = 2/4$, and divides the new range into $[0.5,0.5833]$ and $[0.5833,0.6667]$, each representing "b" and "a" respectively. Since the final symbol is "a", the AC encodes this symbol by choosing any number within $[0.5833,0.6667]$ as an output. By choosing a number which encodes with the fewest number of bits (digits in our example), 0.6, the AC creates its final output. The decoder understands the message length either explicitly in the header of the compressed message or via a special "*end-of-file*" symbol.

The AC iteratively reduces its operating range up to a point when its range is such that the leading digit of the high and low bound are equal. Then, the leading digit can be transmitted. This process, called *renormalization*, enables compression of files of any length on limited precision arithmetic units. Performance improvements of classic AC focus on: using precomputed approximations of arithmetic calculations, replacing division and multiplication with shifting and addition.

An AC encodes a sequence of incoming symbols $s = s_1, s_2, \dots$ using a number of bits equal to source's entropy, $H(s) = -\sum_{s_i} \Pr[s_i] \log_2(\Pr[s_i])$. Hence, for

1 a semi-infinite stream of independent and identically distributed symbols, on a
2 computer with infinite precision arithmetic, the AC is an optimal, entropy coder.

3

4 *2. Arithmetic Encoding of a Min-Distance Point-to-Point Vector*

5 Given an authentication object (L,R,K), it is assumed that light is shed on
6 one of its quadrants, S_i . Next, we assume that the authentication object is
7 partitioned into a grid of $L \times L$ unit squares $U = u(i, j), i = 1 \dots L, j = 1 \dots L$, where each
8 $u(i, j)$ covers the square area within $x \in [i-1, i], y \in [j-1, j]$. Unit areas model the
9 pixels of the digital scan of an authentication object. The resolution of the scan
10 equals $L \times L$. Next, a principal point of a unit $u(x, y)$ is defined as a point Q_u with
11 coordinates (x, y) .

12

13 **Lemma 5. Unit Illumination Likelihood.** Assuming there are κ fibers
14 with exactly one end-point in $S - S_i$, the probability that any unit area
15 $u(x, y) \subset S - S_i$ contains at least one illuminated fiber end-point equals:

16

$$\begin{aligned} \tau(u) &= \Pr[(\exists f = \{A, B\} \in F) A \subset u, B \subset S_i] \quad (16) \\ &= 1 - [1 - \xi(i, u)]^\kappa. \end{aligned}$$

17
18 And

19

$$\begin{aligned} \tau(u) &= \Pr[(\exists f = \{A, B\} \in F) A \subset u, B \subset S_i] = 1 - \Pr[(\neg \exists c \in F) A \subset \\ &u, B \subset S_i] = 1 - (1 - \Pr[A \subset u, B \subset S_i \mid f = \{A, B\}])^\kappa \end{aligned}$$

20

21

22

23 From Equation 7, Equation 16 is concluded. In Section III-B, the
24 expectation for κ is $E[\kappa] = \lambda K/2$ is computed.

1 **Problem 1. Dual Vector Encoding for COA.** Conditioned on the fact that
 2 unit $u \subset S - S_i$ contains an illuminated fiber end-point, a goal is to encode using as
 3 few as possible bits the locations of two other illuminated units v_1 and v_2 relative
 4 to unit u . An additional constraint is that among all illuminated units in $S - S_i$, the
 5 principal points of v_1 and v_2 , Q_1 and Q_2 respectively, are located at two shortest
 6 distances in Euclidean sense from the principal point of u , Q_u . A priority rule is
 7 set so that if a set of units $V, |V| > 1$ are at the same distance with respect to u , the
 8 one with the highest likelihood of illumination: $\text{argmax}_{v \in V} (\tau(v))$ is encoded first.

9

10 Set U as a list of all unit areas in $S - S_i - u$.

11 List of all marked units, $M(u)$, is set to $M(u) = \emptyset$.

12 **do**

13 Find all unit areas $V = \text{argmin}_{v \in U} \|Q_v - Q_u\|$.

14 **do**

15 Find unit area $w = \text{argmax}_{v \in V} \xi(1, v)$.

16 Set AC range for w to $\gamma(w, u)$ (see Eqns.17,18).

17 Set of nodes ordered before w is $M_w(u) = M(u)$.

18 $M(u) = M(u) \cup w$, $V = V - w$, $U = U - w$.

19 **while** $V \neq \emptyset$

20 **while** $U \neq \emptyset$

21 Table 2. ALGORITHM A1.

22

23 The encoding of a unit-to-unit vector is done using an AC, which uses
 24 algorithm A1 to assign a corresponding range on the encoding interval for each
 25 encoding symbol, i.e. each unit $v \subset S - S_i$ different from the source unit u . For

1 each unit v , algorithm A1 assigns a range equal to the probability that v is one of
 2 the two closest illuminated units with respect to the source unit u . This probability
 3 is denoted as $p(v|u)$. In the case when $\kappa \gg 1$ units are expected to illuminate in
 4 $S - S_i$, $p(v|u)$ can be computed as follows:

$$p(v|u) = \tau(v) \prod_{w \in M_v(u)} [1 - \tau(w)] + \quad (17)$$

$$\sum_{w \in M_v(u)} \tau(v)\tau(w) \prod_{z \in M_v(u), z \neq w} [1 - \tau(z)],$$

10 where the set of units $M_v(u)$ is computed as in algorithm A1. For each unit
 11 v , algorithm A1 assigns a range $\gamma(v,u)$ used by the AC to encode v conditioned
 12 on the fact that u has already been encoded. This range is equal to:

$$\gamma(v,u) = \frac{p(v|u)}{\sum_{w \in S - S_i} p(w|u)}. \quad (18)$$

16
 17 Thus, the two nearest illuminated units are encoded by construction near-
 18 optimally (e.g. the encoding is optimal on a processor with infinite precision
 19 arithmetic) because a sequence of symbols is encoded using a number of bits
 20 approximately equal to the entropy of the source:

$$H(u) = - \sum_{v \in S - S_i} \gamma(v,u) \log_2 [\gamma(v,u)]. \quad (19)$$

21
 22
 23 Dual vector encoding is used as a primitive to encode a subset of points in
 24 the overall compression algorithm presented in the Section IV-B. Although the
 25

1 encoding algorithm is near-optimal for the set of assumptions presented in Section
2 IV-A.2, the same set of constraints is not valid for the overall compression goal,
3 hence, the inherent optimality of using arithmetic coding with range allocation via
4 A1 is discussed in Section IV-B.

5

6 **B. Compression of a Point-Subset**

7 The optimization problem of compressing the positions of as many as
8 possible illuminated unit areas using a fixed number of bits is modeled. Consider
9 the following directed complete graph with weighted edges. For each illuminated
10 unit $u \subset S - S_i$, a node n_u is created. A directed edge $e(u, v)$ from node n_u to node
11 n_v is weighted with the optimal length of the codeword that encodes the vector
12 that points to v , $\omega(e(u, v)) = -\log_2 [\gamma(v, u)]$ as in Equation 19, conditioned on the
13 fact that u is already encoded. Lets denote this graph as $G(N, E, \Omega)$, where N , E ,
14 and Ω represent the set of nodes, directed edges, and corresponding weights
15 respectively.

16

17 **Problem 2. Compression of a Point-Subset (CPS).**

18 INSTANCE: Directed, complete, and weighted graph $G(N, E)$ with a non-
19 negative vertex function $\Omega: E \rightarrow R$, positive integer $l_{min} \in Z^+$, positive real number
20 $\Lambda \in R^+$.

21 QUESTION: Is there a subset of $l > l_{min}$ nodes $N^* \subset N$ with a path through
22 them, i.e. a permutation $< n_{\pi(1)}^*, \dots, n_{\pi(l)}^* >$, such that the sum of weights along the
23 path is:

$$\sum_{i=1}^{l-1} \omega(e(n_{\pi(i)}^*, n_{\pi(i+1)}^*)) < \Lambda. \quad (20)$$

Problem 2 models the optimization problem of compressing as many as possible (i.e. l) fiber end-points in an authentication object using a fixed storage (i.e. Λ). This problem is NP-complete as it can be shown that the ASYMMETRIC TRAVELING SALESMAN PROBLEM, ATSP, can be reduced to CPS, $\text{ATSP} \leq_m^p \text{CPS}$, via binary search for Λ . In the remainder of this section, an efficient constructive heuristic A2 is presented that aims at solving this problem. The premier design requirement for the heuristic is fast run-time performance because each certificate of authenticity must be signed separately at a manufacturing line.

First, the distance measure between two nodes in N does not obey the triangle inequality for all nodes. Intuitively, the encoding procedure from Section IV-A encodes vectors in $S - S_i$ using a number of bits proportional to the likelihood that a certain unit is one of the two closest illuminated points. Hence, units farther from the source node are encoded with significantly longer codewords as they are unlikely to occur, which renders shortcuts to these nodes in the solution route highly undesirable.

Theorem 2. The distance measure ω does not universally obey the triangle inequality:

$$\omega(e(u, v)) + \omega(e(v, w)) \geq \omega(u, w).$$

1 For simplicity, assume that $(\forall u \in S - S_i) t = \tau(u) = \text{const.}$, then u , v , and w
 2 are positioned along the same line in $S - S_i$. The Euclidean distances $\|u - v\|$,
 3 $\|v - w\|$, and $\|u - w\|$ are a , b , and $a+b$ respectively. The triangle inequality
 4 implies that $f(u, v, w) = \log_2 [\gamma(w, u)] - \log_2 [\gamma(v, u)] - \log_2 [\gamma(w, v)] \geq 0$. From
 5 Equations 17 and 18, the following can be computed:

$$f(a, b, t) = 2ab\pi \log_2(1-t) + \log_2 \frac{t}{1-t} - \log_2 \frac{(1-t)^2 + (a^2 + b^2)\pi t(1-t) + a^4 b^4 \pi^2 t^2}{1 + [(a+b)^2 \pi - 1]t} \quad (21)$$

10 and show that for $ab\pi t \gg 1$, the triangle inequality does not hold, i.e.,
 11 $f(a, b, t) < 0$.

12 The best approximation algorithm for ATSP where the triangle inequality
 13 holds, yields solutions at most $\log(|N|)$ times worse than the optimal.
 14 Alternatively, to the best knowledge of the authors, approximation algorithms for
 15 ATSP variants where the triangle inequality does not hold, have not been
 16 developed. In the general case, when the distance metric function ω is arbitrary,
 17 the ATSP problem is NPO-complete, i.e. there is no good approximation algorithm
 18 unless $P = NP$. On the other hand, approximation algorithms for variants of TSP
 19 which satisfy a scaled version of the triangle inequality:
 20 $\mu(\omega(e(u, v)) + \omega(e(v, w))) \geq \omega(u, w)$, $\mu > 1$ can be solved with a worst case result
 21 $(3\mu + 1)\mu/2$ times worse than the optimal solution. Distance metric ω does not
 22 follow this constraint, hence, a heuristic for Problem 2 is developed without a
 23 worst-case guarantee. In addition, we aim for as good as possible performance of
 24 the heuristic on the average, rather than a worst-case guarantee. Authentication
 25

1 object instance which cannot be compressed satisfactorily can be disposed.
 2 Likelihood of this event should be small, less than one in a million.

3
 4 CONSTRUCTIVE PHASE

5 Set of edges $E' = \{\text{argmin}_e(\omega(a, b), \omega(b, a)) \mid (\forall a, b) \subset N\}$.

6 Set of subpaths P is selected as a set of shortest K edges

7 in E' s.t. $\sum_{i=1}^K \omega(e_i) \leq \Lambda$ sorted by ω .

8 Denote the weight of the shortest edge in E as ω_{\min} .

9 **for each** path $p_i \subset P, i = 1..K-1$

10 **for each** path $p_j \subset P, j = i+1..K$

11 **if** p_i and p_j have a common source-destination node

12 Concatenate p_i and p_j as $p_i = p_i \mid p_j$.

13 Remove p_j from P .

14 Denote source and destination nodes of a path $p_i \subset P$

15 as s_i and d_i respectively.

16 **for each** path $p_i \subset P, i = 1..K$

17 Find all shortest paths $q(i, j)$ from s_i to any $d_j, j \neq i$.

18 **while** $|P| < \text{maxP}$

19 $(p_i, p_j) = \text{argmin}_{q(i, j)} \sum_{e \in \{p_i \mid q(i, j) \mid p_j\}} \frac{\omega(e)}{\|(p_i \mid q(i, j) \mid p_j)\|}$.

20 Concatenate $p_i = p_i \mid q(i, j) \mid p_j$ and remove p_j from P .

21 Find exhaustively a concatenation $p_h = p_1 \mid \dots \mid p_{\text{maxP}}$ s.t.

22 $M(p_h) \{ \sum_{e \in p_h} \omega(e) < \Lambda \text{ and } |p_h| \text{ is maximal} \}$.

23 **reroute**(p_h)

24 **reroute**(p_h)

25 $p_{\text{best}} = p_h$

```

1   for each edge  $e(s_i, d_i) \subset p_h, i = 1, \dots, |p_h| - 1$ 
2       for each node pair  $(d_i, s_j) \subset p_h, j = i + 2, \dots, |p_h| - 1$ .
3           Find shortest path  $q(i, j)$  via nodes in  $N - p_h$ .
4           if path  $e_1, \dots, e_i | q(i, j) | e_j, \dots, e_{|p_h|}$  has a better metric
5                $M(p_h)$  then  $p_{\text{best}}$ 
6               then  $p_{\text{best}} = p_h$ .
7
8   GREEDY ITERATIVE IMPROVEMENT
9
10  repeat  $I$  times
11      Contract  $p_h$  so that  $\sum_{e \in p_h} \omega(e) \leq \rho \Lambda$ , where  $\rho$  is a
12          contraction factor, randomly chosen from  $\rho \in \{0.4, 0.8\}$ .
13
14      Denote nodes  $n_0$  and  $n_l$  as the first and last node in  $p_h$ .
15      while  $\sum_{e \in p_h} \omega(e) \leq \Lambda$ 
16          Among edges that have  $n_0$  or  $n_l$  as destination or
17          source respectively, find edge  $e$  with minimal weight.
18
19          Concatenate  $e$  to  $p_h$ .
20
21          reroute( $p_h$ )

```

TABLE 3. ALGORITHM A2.

The rationale behind using the distance metric ω from Section IV-A is based on an assumption that a good solution succeeds to traverse each node on its route via the two closest neighboring nodes. Hence, in the scope of Problem 2, the used metric is optimal only if the best solution found satisfies this property. If the final solution does not have this property, the optimality of encoding a single vector is dependent upon the distribution of weights of the edges in the solution.

1 The developed heuristic A2 has two stages: a constructive and an iterative
2 improvement phase. The constructive phase follows a greedy heuristic which
3 builds the initial solution. Initially, A2 identifies a set of dominating edges E' . For
4 each pair of edges, $e(u,v)$, $e(v,u)$, between nodes u,v , A2 selects only the shorter
5 of the two and stores it in E' . Next, a set P of initial subpaths is created by
6 sorting the edges in E' and selecting the top K shortest edges whose weights sum
7 up as close as possible to Λ . The first and last node in a path p_i are denoted as s_i
8 and d_i , respectively. In the next step, A2 concatenates subpaths from P iteratively
9 in the increasing order of their weights: at any point, the pair of shortest subpaths
10 p_i, p_j which have a common source-destination node $d_i = s_j$, is concatenated until
11 all possible connections are established. In the unlikely case when $|P|=1$, the
12 optimal solution is found and the search is stopped. Else, all single-edge subpaths
13 are removed from P . Then, using Dijkstra's algorithm, A2 finds all shortest paths
14 between each destination tail d_i of each subpath p_i in P and source tails of all
15 other subpaths, $s_j, i=1\dots|P|, i \neq j$. The shortest paths are routed via nodes which
16 are not in P . The shortest path is denoted between s_i and d_j as $q(i,j)$. In another
17 greedy step, A2 sorts all concatenations $p_i | q(i,j) | p_j$ according to their
18 weight/node count ratio. In increasing order of this metric, A2 continues
19 concatenating subpaths in P via nodes in $N - P$ until the total number of
20 remaining paths is $|P| = \text{maxP}$ (usually $\text{maxP} = 9$). The remaining paths are
21 concatenated using an exact algorithm which finds a path p_h with the optimal
22 metric: maximal cardinality and a sum of weights smaller than Λ . In the final step,
23 a rerouting procedure browses all the nodes in P , and using the Dijkstra algorithm
24 tries to find shortest paths to other nodes in P via the remaining nodes in E . The
25 same procedure also tries to find a better ending tail than the one that exists in p_h .

1 For each reroute, A2 checks whether the new reroute has a better metric than the
2 current, best path p_h .

3 Figure 11 is an example of an instance of an authentication object
4 (512,0.4·512,256) is shown with $\kappa=88$ nodes. A2 returned the path illustrated
5 with bold lines. The path is such that its sum of weights is smaller than $\Lambda=512$.
6 To document the path, 12.11 bits per point is used.

7 In the iterative improvement phase, we repeat several rounds of the
8 following loop. In the first step, A2 contracts the currently best found path p_{best}
9 into p_h , so that $|p_h|$ is maximal and the sum of weights along p_h is smaller than a
10 fraction of $\rho\Lambda$. The contraction parameter ρ is randomly selected in each
11 iteration within $\rho \in \{0.4, 0.8\}$. Nodes n_0 and n_l are denoted as the first and last
12 node in p_h . While the sum of weights in p_h is smaller than Λ , among edges that
13 have n_0 or n_l as destination or source respectively, we find an edge e with
14 minimal weight and concatenate it to p_h . When the new candidate path p_h is
15 created, it is adopted as the best solution if its metric is better than the metric of
16 the best path created so far. As a last step of the iterative improvement loop, A2
17 performs the rerouting procedure previously described.

18 In order to fit the run-time of A2 for a particular authentication object
19 (L, R, K) class within one second, the improvement loop is repeated $I = \{100, 10000\}$
20 times. In general, the worst-time complexity of A2 is $O(|N|^3 \log |N|)$ as multi-
21 source shortest paths are computed via the Dijkstra algorithm. In an
22 implementation that uses the Floyd-Warshall algorithm to compute all pairs
23 shortest paths, the complexity of A2 can be reduced to $O(|N|^3)$. Although the
24 graph is originally complete, by removing edges with high weights, we create a
25

1 sparse graph, where Johnson's algorithm for all-pairs shortest paths yields
2 $O(|N|^2 \log |N| + |N||E|)$.

3

4 **V. Empirical Evaluation**

5 The discussion in this section shows how authentication object (L,R,K)
6 parameters impact the performance of the algorithm A.2. Figure 11 illustrates a
7 solution to a single instance of the problem, an authentication object
8 (512,0.4·512,256). The scanning grid to $L = 512$ scanning cells. The figure
9 depicts the case when the lower left quadrant of the authentication object is
10 illuminated. Graph $G(N, E)$, built using the corresponding illuminated fiber end-
11 points, is illustrated with medium bold lines. Only the top ten shortest edges
12 starting from each of the $\kappa = 88$ nodes in the graph is shown. The resulting path
13 shown in the figure using bold lines, consists of 41 nodes. The sum of weights
14 along path's edges is smaller than the storage limit: $\Lambda = 512$ bits. The path is
15 compressed using 12.11 bits per fiber end-point (b/fep). Storing the data without
16 compression would require $41 \cdot 18 = 738$ bits, which results in a compression ratio
17 of 0.61. The compression ratio is defined as a ratio of the size of the compressed
18 message vs. the original message size.

19

20 **VI. A Design Objective for a COA System**

21 A goal of the certificate of authenticity designer is to maximize the cost of
22 forgery ς_f using a bounded manufacturing cost ς_m . Several parameters may
23 impact ς_m . For brevity and simplicity, three parameters are discussed:

24 the total length of fiber $RK \leq \Phi$,

25 the scanning tolerance ζ , and

1 the barcode storage Λ .

2 System performance is optimized by limiting the number of trials available
3 to the adversary for accurate positioning of a sufficient subset of the signed fiber
4 end-points (Section VI-A) and by selecting the system parameters $\{R_*, K_*, \zeta\}$ so that
5 expected forging cost $\zeta_f(A2)$ is maximized (Section VI-B).

6

7 A. Limiting the Number of Adversarial Trials

8 Consider a compression scheme C which stores G out of the κ
9 illuminated fiber end-points in a Λ -limited storage. In general, when forging a
10 certificate of authenticity, the adversary can use all κ fibers to try to place at least
11 $G\zeta$ of them accurately at their corresponding locations. Cost of forging a
12 certificate of authenticity greatly depends upon the number of available trials.
13 Here, a technique is proposed which aims at reducing the number of adversarial
14 trials, K_T , by detecting anomalous distribution of fibers around the signed fiber
15 end-points during verification.

16

17

18 **ISSUING A COA INSTANCE**

19 Scan for a set N of κ points, illuminated when light is shed on S_i .

20 Using Λ bits, compress a subset $P \subset N$, with $G = |P| \leq \kappa$.

21 Find a subset of units $U \subset S - S_i$, such that

22
$$(\forall u_i \in U)(\forall p_j \in P) \min(\|u_i - p_j\|) < \varepsilon_1.$$

23
$$\varepsilon_2 = |N \cap U| - G, \quad K_T = G + \varepsilon_2.$$

24 Sign P, ε_2 and the associated information (see Section 2).

25 **VERIFYING A COA INSTANCE**

```

1 Extract  $P, \varepsilon_2$  from signature.
2 Find a subset of units  $U \subset S - S_i$ , such that
3  $(\forall u_i \in U)(\forall p_j \in P) \min(\|u_i - p_j\|) < \varepsilon_1$ .
4 Scan for a set  $N'$  of  $\kappa'$  points, illuminated when light is shed on  $S_i$ .
5 if  $|N' \cap U| > K_T$  then COA instance is invalid,
6 elseif  $|N' \cap P| \geq G\zeta$  then COA instance is valid,
7 else COA instance is invalid.

```

TABLE 4. ALGORITHM A3

The certificate of authenticity issuer and verifier repeat their parts of the algorithm A3 for each authentication object quadrant S_i . The issuer initially scans the authentication object instance and collects information about the set of points N which illuminate when S_i is lit up. Next, using the available Λ bits, it compresses the largest subset $P \subset N$, $|P|=G$ returned by A2. Then, A3 finds a subset $U \subset S - S_i$, such that the Euclidean distance between each unit $u_i \in U$ and its closest unit $p_j \in P$ is at most ε_1 . Subset U of units represents an ε_1 -neighborhood of P . Then, the issuer counts the number K_T of points in N that exist in U . Since, K_T has to be greater than G to prevent false negatives, the issuer stores along with P , the difference $\varepsilon_2 = K_T - G$ in the message m , which is later signed using the private key of the issuer (see Section II). Using the public key of the issuer, the verifier extracts from the attached signature the compressed point subset P and ε_2 and recreates the corresponding ε_1 -neighborhood, U . Then, the verifier scans the authentication object instance for the set of illuminated fibers N' when S_i is lit up. It announces that the instance is authentic by checking that

1 the number of common points in U and N' is at most $G + \varepsilon_2$ and that the number
2 of common points in N' and P is at least $G\zeta$.

3 By storing ε_2 in the signature, the adversary is imposed to use at most
4 $K_T = G + \varepsilon_2$ trials that position fibers in the ε_1 -neighborhood of P . The adversary's
5 goal is to place at least $G\zeta$ fiber end-points from P accurately, hence, the
6 adversary can afford $G(1 - \zeta) + \varepsilon_2$ misplacements located in the ε_1 -neighborhood
7 of P during the forgery process. It is expected that each trial, targeting a point p_i ,
8 if unsuccessful, ends up in the ε_1 -neighborhood of p_i . By increasing ε_1 , the
9 verifier can identify possible misplacements over a larger neighborhood; however,
10 this also increases the expectation for ε_2 - a value that the certificate of
11 authenticity designer wants to keep as low as possible.

12 Below, an empirical design methodology is shown which adopts a given
13 $\varepsilon_1 = \text{const.}$, and then seeks to maximize the main objective $\varsigma_f(A2)$ from the
14 perspective of several certificate of authenticity parameters.

15

16 B. Designing a COA System

17 **Problem 3. A Design Objective for a COA System.** For a given
18 compression algorithm A2, fixed $RK \leq \Phi$, ζ , ε_1 , and Λ , find a cut $\{R_*, K_*\}$ of the
19 available fiber which maximizes:

20

$$21 \{R_*, K_*\} = \arg \max_{\{R, K | RK \leq \Phi\}} \varsigma_f(A2, R, K), \quad (22)$$

22
23 where ς_f is defined in Lemma 2. Note that the number of trials κ in Eqn.2
24 equals K_T as presented in Subsection VI-A. Compression performance G in
25 Equation 2 depends upon the efficacy of A2.

1 Figure 12 is a graphical representation of a certificate of authenticity design
2 for optimized cost effectiveness. The abscissa quantifies fiber length R relative to
3 L , while the ordinate shows the number of fibers K . The bar illustrates the log -
4 cost of forgery $\log_{10}(\zeta_f(A2, R, K))$ with a constraint limit $\Lambda = 512$ bits and a set of
5 fixed parameters: $\zeta = 0.9$, $\varepsilon_1 = 8$, and $\nu = 0.8$. The figure also illustrates the quality
6 of solutions obtained for all cuts of a fixed length fiber $RK = \Phi = 100L$.

7 A simple empirical technique may be used that searches for the best fiber
8 cut $\{R_*, K_*\}$. The search procedure is illustrated using Figure 12. The abscissa and
9 the ordinate represent the values of R and K respectively. The bar denotes the
10 expected log-cost of forging an certificate of authenticity instance,
11 $\log_{10}(\zeta_f(A2, RK))$. The cost is given with respect to R and K , and for a fixed set
12 of parameters: $\Lambda = 512$, $\zeta = 0.9$, $\varepsilon_1 = 8$, and $\nu = 0.8$. The diagram in Figure 12 was
13 computed empirically. A2 is applied to 500 randomly generated certificate of
14 authenticity $(512, R, K)$ instances with each combination of
15 $R = \{0.05L, 0.10L, \dots, 0.45L\}$ and $K = \{80, 96, \dots, 192, 256, 384, 512, 768, 1024\}$. The
16 expected compression performance for each point in the remaining portion of the
17 $\{R, K\}$ -space was obtained by interpolating the empirical results. From Figure 12,
18 the best fiber cut can be found in the neighborhood of $K_* \approx 900$ and $R_* \approx 0.1L$.
19 This result points to the fact that for the selected design environment, a cross-
20 shaped certificate of authenticity is the best option. Note that careful selection of
21 the fiber cut resulted in an order of magnitude improvement in the forgery cost
22 with respect to a randomly selected point on $RK = \Phi$. The empirical principles
23 used in this example, can be applied to search for a near-optimal parameter set for
24 different certificate of authenticity environments and manufacturing constraints.

1 Fig. 13 illustrates an example computing device 1300 within which the
2 described systems and methods can be either fully or partially implemented.
3 Computing device 1300 is only one example of a computing system and is not
4 intended to suggest any limitation as to the scope of the use or functionality of the
5 invention.

6 Computing device 1300 can be implemented with numerous other general
7 purpose or special purpose computing system environments or configurations.
8 Examples of well known computing systems, environments, and/or configurations
9 that may be suitable for use include, but are not limited to, personal computers,
10 server computers, thin clients, thick clients, hand-held or laptop devices,
11 multiprocessor systems, microprocessor-based systems, set top boxes,
12 programmable consumer electronics, network PCs, minicomputers, mainframe
13 computers, gaming consoles, distributed computing environments that include any
14 of the above systems or devices, and the like.

15 The components of computing device 1300 can include, but are not limited
16 to, processor 1302 (e.g., any of microprocessors, controllers, and the like), system
17 memory 1304, input devices 1306, output devices 1308, and network devices
18 1310.

19 Computing device 1300 typically includes a variety of computer-readable
20 media. Such media can be any available media that is accessible by computing
21 device 1300 and includes both volatile and non-volatile media, removable and
22 non-removable media. System memory 1304 includes computer-readable media
23 in the form of volatile memory, such as random access memory (RAM), and/or
24 non-volatile memory, such as read only memory (ROM). A basic input/output
25 system (BIOS), containing the basic routines that help to transfer information

1 between elements within computing device 1300, such as during start-up, is stored
2 in system memory 1304. System memory 1304 typically contains data and/or
3 program modules that are immediately accessible to and/or presently operated on
4 by processor 1302.

5 System memory 1304 can also include other removable/non-removable,
6 volatile/non-volatile computer storage media. By way of example, a hard disk
7 drive may be included for reading from and writing to a non-removable, non-
8 volatile magnetic media; a magnetic disk drive may be included for reading from
9 and writing to a removable, non-volatile magnetic disk (e.g., a “floppy disk”); and
10 an optical disk drive may be included for reading from and/or writing to a
11 removable, non-volatile optical disk such as a CD-ROM, DVD, or any other type
12 of optical media.

13 The disk drives and their associated computer-readable media provide
14 non-volatile storage of computer-readable instructions, data structures, program
15 modules, and other data for computing device 1300. It is to be appreciated that
16 other types of computer-readable media which can store data that is accessible by
17 computing device 1300, such as magnetic cassettes or other magnetic storage
18 devices, flash memory cards, CD-ROM, digital versatile disks (DVD) or other
19 optical storage, random access memories (RAM), read only memories (ROM),
20 electrically erasable programmable read-only memory (EEPROM), and the like,
21 can also be utilized to implement exemplary computing device 1300. Any number
22 of program modules can be stored in system memory 1304, including by way of
23 example, an operating system 1320, application programs 1328, and data 1332.

24 Computing device 1300 can include a variety of computer-readable media
25 identified as communication media. Communication media typically embodies

1 computer-readable instructions, data structures, program modules, or other data in
2 a modulated data signal such as a carrier wave or other transport mechanism and
3 includes any information delivery media. The term “modulated data signal” refers
4 to a signal that has one or more of its characteristics set or changed in such a
5 manner as to encode information in the signal. By way of example, and not
6 limitation, communication media includes wired media such as a wired network or
7 direct-wired connection, and wireless media such as acoustic, RF, infrared, and
8 other wireless media. Combinations of any of the above are also included within
9 the scope of computer-readable media.

10 A user can enter commands and information into computing device 1300
11 via input devices 1306 such as a keyboard and a pointing device (e.g., a “mouse”).
12 Other input devices 1306 may include a microphone, joystick, game pad,
13 controller, satellite dish, serial port, scanner, touch screen, touch pads, key pads,
14 and/or the like. Output devices 1308 may include a CRT monitor, LCD screen,
15 speakers, printers, and the like.

16 Computing device 1300 may include network devices 1310 for connecting
17 to computer networks, such as local area network (LAN), wide area network
18 (WAN), and the like.

19 Although the invention has been described in language specific to structural
20 features and/or methodological steps, it is to be understood that the invention
21 defined in the appended claims is not necessarily limited to the specific features or
22 steps described. Rather, the specific features and steps are disclosed as preferred
23 forms of implementing the claimed invention.

24
25